＊ 潮位の調和分解：実習ノート【１年生フレッシュマンセミナ】

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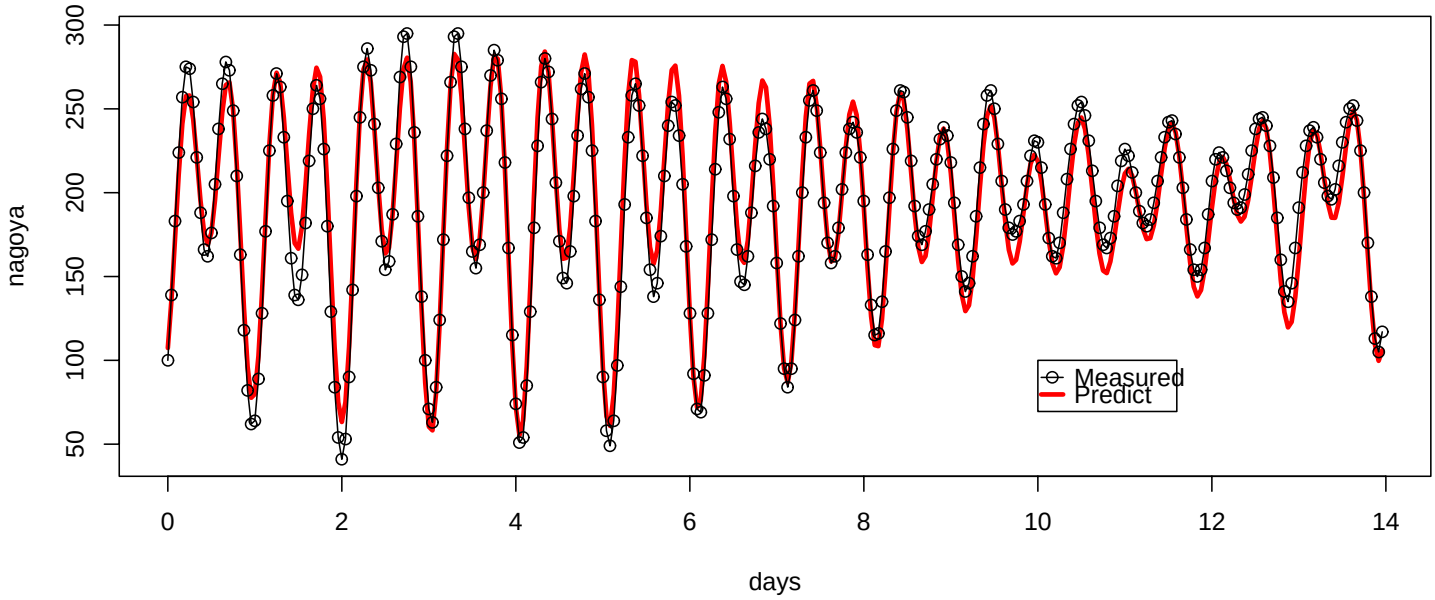

（このセミナでは、以下の内容を『**直感的に理解**』しながら実習を行う．
本格的な理解は，上級学年でのお楽しみに，. . .
なお，以下の内容をベクトルと内積で理解できるようになれば，本質的理解は完了です．）

＊ $y = z_0 + A_1 \cos(\omega_1 t) + B_1 \sin(\omega_1 t) + A_2 \cos(\omega_2 t) + B_2 \sin(\omega_2 t) + \ldots$

$$\zeta_0 = \frac{1}{T_0}\int_0^{T_0} y\,dt\,; \quad \alpha_j = \frac{2}{T_0}\int_0^{T_0} y\cos(\omega_j t)\,dt\,; \quad \beta_j = \frac{2}{T_0}\int_0^{T_0} y\sin(\omega_j t)\,dt$$

1) Fourier expansion (efficient by FFT method): $\omega_k = 2\pi k/T_0$

$$z_0 = \zeta_0\,; \quad A_k = \alpha_k\,; \quad B_k = \beta_k$$

$$\frac{1}{T_0}\int_0^{T_0} \cos(\omega_i t)\cos(\omega_j t)\,dt = \begin{cases} 1/2 & (\omega_i = \omega_j) \\ 0 & (\omega_i \neq \omega_j) \end{cases}$$

$$\frac{1}{T_0}\int_0^{T_0} \sin(\omega_i t)\sin(\omega_j t)\,dt = \begin{cases} 1/2 & (\omega_i = \omega_j) \\ 0 & (\omega_i \neq \omega_j) \end{cases}$$

$$\frac{1}{T_0}\int_0^{T_0} \cos(\omega_i t)\sin(\omega_j t)\,dt = 0$$

2) Harmonic decomposition (by least square method): $\omega_k \neq 2\pi k/T_0$

$$\{\, z_0\,;\, A_k\,;\, B_k \,\} = \{\, \zeta_0\,;\, \alpha_k\,;\, \beta_k \,\}\{M_{i,j}\}$$

```
R : Copyright 2003, The R Development Core Team
Version 1.7.1  (2003-06-16)

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You are welcome to redistribute it under certain conditions.
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R is a collaborative project with many contributors.
Type `contributors()' for more information.

Type `demo()' for some demos, `help()' for on-line help, or
`help.start()' for a HTML browser interface to help.
Type `q()' to quit R.

> date()
[1] "Mon May 30 18:29:55 2005"

## 0 ##
> cos(2*pi*1:24/24) -> cos.1
> cos.1
 [1]  9.659258e-01  8.660254e-01  7.071068e-01  5.000000e-01  2.588190e-01  6.123234e-17
 [7] -2.588190e-01 -5.000000e-01 -7.071068e-01 -8.660254e-01 -9.659258e-01 -1.000000e+00
[13] -9.659258e-01 -8.660254e-01 -7.071068e-01 -5.000000e-01 -2.588190e-01 -1.836970e-16
[19]  2.588190e-01  5.000000e-01  7.071068e-01  8.660254e-01  9.659258e-01  1.000000e+00
> plot(cos.1)

> cos(2*pi*0:23/24) -> cos.1
> points(cos.1, col="red")
> points(0:23, cos.1, pch=19)

## 1 ##
> nami <- .3 *cos.1 - .4
> points(nami, pch=6)

> mean(nami)
[1] -0.4
> rep(1, 24)
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
> sum(rep(1, 24) * nami)/24
[1] -0.4
> sum( cos.1 * cos.1 )/24
[1] 0.5
> sum(2* cos.1 * nami)/24
[1] 0.3

## 2 ##
> cos(2*pi*0:23/24 * 2) -> cos.2
> plot(cos.2)
> sum(2* cos.2 * cos.1)/24
[1] -2.312965e-16
> # this means zero. it's just a problem of precision.
> sum(2* cos.2 * cos.2)/24
[1] 1

## 3 ##
> t <- seq(0, 24, by=1/60)
> length(t)
[1] 1441
> cos(2*pi*t[-1]/24) -> cos.1.minutes  # or t[-1441]
> length(cos.1.minutes)
[1] 1440
> plot(cos.1.minutes)
> sum(2* cos.1.minutes * cos.1.minutes )/1440
[1] 1
```

２４時間 ～　１時から始めるか？　０時から始めるか？
いずれにせよ，１時間の間隔であれば，２４個で計る．
０時から２４時は２５個！（周期性を検討する上で，ダメ！）

平均の演算 ＝ 総和／個数
という単純な枠組みに，重みつき総和と考えて，重みが等しい場合が，平均と考えよ．

余弦関数を重みとすれば，...

```
## 4 ##
> sin(2*pi*0:23/24) -> sin.1      # or sin(2*pi*(1:24 - 1)/24)
> sin(2*pi*0:23/24 * 2) -> sin.2  # or sin(2*pi*(1:24 - 1)/12)
> sum(2* cos.1 * sin.1 )/24
[1] -1.850372e-17
> sum(2* sin.1 * sin.1 )/24
[1] 1
> sum(2* cos.1 * sin.2 )/24
[1] -1.480297e-16
> sum(2* sin.2 * sin.2 )/24
[1] 1
> sum(2* cos.2 * sin.2 )/24
[1] 1.156482e-16

## 5 ##
> y <- cos(2*pi*0:23/24 + pi/4)
> plot(y)
> points(cos(2*pi*0:23/24) * cos(pi/4) - sin(2*pi*0:23/24) * sin(pi/4), pch="X")
> c(cos(pi/4), -sin(pi/4))
[1]  0.7071068 -0.7071068
> sum(2 * cos.1 * y)/24
[1] 0.7071068
> sum(2 * sin.1 * y)/24
[1] -0.7071068

## 6 ##
[ visit the web page of Japan Oceanographic Data Center at http://www.jodc.go.jp/ ]

MA22,03/01/01,100,139,183,224,257,275,274,254,221,188,166,162,1[illegible]265,278,273,249,210,163,118,082,062
MA22,03/01/02,064,089,128,177,225,258,271,263,233,19[illegible]264,256,226,180,129,084,054
MA22,03/01/03,041,053,090,142,198,245,275,286,27[illegible]295,275,236,186,138,100
MA22,03/01/04,071,063,084,124,172,222,266,293[illegible]279,256,218,167,115
MA22,03/01/05,074,051,054,085,129,179,228,26[illegible]271,257,225,183,136
MA22,03/01/06,090,058,049,064,097,144,193,[illegible],252,234,205,168
MA22,03/01/07,128,092,071,069,091,128,172,[illegible]244,238,220,192
MA22,03/01/08,158,122,095,084,095,124,162,[illegible]238,242,236,221
MA22,03/01/09,195,163,133,115,116,135,165,[illegible],220,232,239,234
MA22,03/01/10,218,194,169,150,141,146,162,1[illegible]3,193,207,222,231
MA22,03/01/11,230,215,193,173,162,161,170,188,[illegible]167,173,186,204,219
MA22,03/01/12,226,222,212,200,189,182,180,184,19[illegible]56,154,150,154,167,187
MA22,03/01/13,207,220,224,221,213,203,194,190,191,19[illegible]59,185,160,141,135,146,167
MA22,03/01/14,191,212,228,237,239,233,220,206,198,196,202,21[illegible],252,243,225,200,170,138,113,105,117

100 139 183 224 257 275 274 254 221 188 166 162 1[illegible]265 278 273 249 210 163 118 082 062
064 089 128 177 225 258 271 263 233 195 [illegible]264 256 226 180 129 084 054
041 053 090 142 198 245 275 286 273 [illegible]295 275 236 186 138 100
071 063 084 124 172 222 266 293 2[illegible]279 256 218 167 115
074 051 054 085 129 179 228 266[illegible]271 257 225 183 136
090 058 049 064 097 144 193 23[illegible]252 234 205 168
128 092 071 069 091 128 172 2[illegible]244 238 220 192
158 122 095 084 095 124 162 2[illegible]238 242 236 221
195 163 133 115 116 135 165 19[illegible]220 232 239 234
218 194 169 150 141 146 162 186[illegible]193 207 222 231
230 215 193 173 162 161 170 188 2[illegible]167 173 186 204 219
226 222 212 200 189 182 180 184 194 2[illegible]56 154 150 154 167 187
207 220 224 221 213 203 194 190 191 199 2[illegible]59 185 160 141 135 146 167
191 212 228 237 239 233 220 206 198 196 202 216 2[illegible]252 243 225 200 170 138 113 105 117

> scan() -> nagoya
1: (paste here)
337:
Read 336 items
> plot(nagoya)
> days <- (1:(24*14) - 1)/24
> plot(days, nagoya, type="o")
```

このようなデータの形式を
エディタにて，以下のような
形式に変更せよ！

これをコピーして，scan ... の後で，
paste する．

より本格的なデータ入力には，データを
ファイルに保存してから，read.table
などを用いる（ここでは，簡便に行う）．

```
## 7 ##
> cos(2*pi*(1:(24*14) - 1)/24) -> cos.1
> sin(2*pi*(1:(24*14) - 1)/24) -> sin.1
> cos(2*pi*(1:(24*14) - 1)/12) -> cos.2
> sin(2*pi*(1:(24*14) - 1)/12) -> sin.2
> 24*14
[1] 336
> 24*14 /2
[1] 168
> z0 <- mean(nagoya)
> a1 <- sum(2 * cos.1 * nagoya)/336
> b1 <- sum(2 * sin.1 * nagoya)/336
> a2 <- sum(2 * cos.2 * nagoya)/336
> b2 <- sum(2 * sin.2 * nagoya)/336
> c( z0, a1, b1, a2, b2 )
[1] 193.482143 -30.888663 -12.575485  -8.759809 -26.126645
> lines(days, z0 + a1 * cos.1 + b1 * sin.1 + a2 * cos.2 + b2 * sin.2, col="blue")
```

１日に１回の振動と２回の振動を考えて，...

```
## 8 ##
cos.K1 <- cos(2*pi*(1:(24*14) - 1)/23.93)
sin.K1 <- sin(2*pi*(1:(24*14) - 1)/23.93)

cos.O1 <- cos(2*pi*(1:(24*14) - 1)/25.82)
sin.O1 <- sin(2*pi*(1:(24*14) - 1)/25.82)

cos.M2 <- cos(2*pi*(1:(24*14) - 1)/12.42)
sin.M2 <- sin(2*pi*(1:(24*14) - 1)/12.42)

cos.S2 <- cos(2*pi*(1:(24*14) - 1)/12.00)
sin.S2 <- sin(2*pi*(1:(24*14) - 1)/12.00)

a.K1 <- sum(2 * cos.K1 * nagoya)/336
b.K1 <- sum(2 * sin.K1 * nagoya)/336

a.O1 <- sum(2 * cos.O1 * nagoya)/336
b.O1 <- sum(2 * sin.O1 * nagoya)/336

a.M2 <- sum(2 * cos.M2 * nagoya)/336
b.M2 <- sum(2 * sin.M2 * nagoya)/336

a.S2 <- sum(2 * cos.S2 * nagoya)/336
b.S2 <- sum(2 * sin.S2 * nagoya)/336

lines(days, z0 + a.K1 * cos.K1 + b.K1 * sin.K1 +
                 a.O1 * cos.O1 + b.O1 * sin.O1 +
                 a.M2 * cos.M2 + b.M2 * sin.M2 +
                 a.S2 * cos.S2 + b.S2 * sin.S2,  col="magenta")
```

１日に１回の潮汐の周期は, 厳密に２４時間ではない！
付表を参考にして，...

```
## 9 ##
> sum(2 * cos.K1 * cos.K1)/336 # !?
[1] 1.002918
> sum(2 * sin.K1 * cos.K1)/336 # !?
[1] -1.532324e-05
> sum(2 * cos.M2 * cos.K1)/336 # !?
[1] 0.003239341
> lm(nagoya ~ cos.K1 + sin.K1 + cos.O1 + sin.O1 +
              cos.M2 + sin.M2 + cos.S2 + sin.S2) -> nagoya.fit
> coef(nagoya.fit)
(Intercept)      cos.K1      sin.K1      cos.O1      sin.O1      cos.M2      sin.M2      cos.S2      sin.S2
 193.645343  -32.184377   -8.203511   -5.446769   18.871379  -27.024005   53.807887  -21.632820  -14.974183

> rbind(coef(nagoya.fit), c(z0, a.K1, b.K1, a.O1, b.O1, a.M2, b.M2, a.S2, b.S2))
     (Intercept)    cos.K1    sin.K1    cos.O1   sin.O1    cos.M2   sin.M2    cos.S2    sin.S2
[1,]    193.6453 -32.18438 -8.203511 -5.446769 18.87138 -27.02401 53.80789 -21.63282 -14.97418
[2,]    193.4821 -31.41825 -7.713845 -6.136877 18.78467 -25.43807 54.74563 -20.64954 -18.24643

> lines(days, predict(nagoya.fit), col="red")
```

しかし，この場合には，問題が生じる.

```
## 10 ##
> library(chron) # Not Necessary, ...
> chron("01/01/2004") - chron("12/31/2003")
[1] 1
> chron("04/01/2004") - chron("12/31/2003")
Time in days:
[1] 92
> chron("06/01/2005") - chron("12/31/2003")
Time in days:
[1] 518
```

日付けの計算もできるよ！
（追加パッケージを使用する練習）

```
MA22 03/01/15 143 174 202 223 235 238 230 215 197 179 168 171 186 205 222 233 231 217 193 163 128 096 078 079
MA22 03/03/22 160 122 105 113 144 189 235 270 287 281 250 201 147 104 079 076 094 133 181 225 257 271 262 235

## 11 ##
c.K1 <- function (day) cos(2*pi*(0:23 + 24*(day - 1))/23.93)
s.K1 <- function (day) sin(2*pi*(0:23 + 24*(day - 1))/23.93)

c.O1 <- function (day) cos(2*pi*(0:23 + 24*(day - 1))/25.82)
s.O1 <- function (day) sin(2*pi*(0:23 + 24*(day - 1))/25.82)

c.M2 <- function (day) cos(2*pi*(0:23 + 24*(day - 1))/12.42)
s.M2 <- function (day) sin(2*pi*(0:23 + 24*(day - 1))/12.42)

c.S2 <- function (day) cos(2*pi*(0:23 + 24*(day - 1))/12.00)
s.S2 <- function (day) sin(2*pi*(0:23 + 24*(day - 1))/12.00)

water.level <- z0 + a.K1 * c.K1(15) + b.K1 * s.K1(15) + a.O1 * c.O1(15) + b.O1 * s.O1(15) +
                    a.M2 * c.M2(15) + b.M2 * s.M2(15) + a.S2 * c.S2(15) + b.S2 * s.S2(15)
plot(0:23, water.level, type="l", xlim=c(0,24))

water.level.fit <- matrix(
       c(c.K1(15), s.K1(15), c.O1(15), s.O1(15), c.M2(15), s.M2(15), c.S2(15), s.S2(15)),
        ncol=8) %*% coef(nagoya.fit)[-1] + coef(nagoya.fit)[1]
points(0:23, out, col="red")

points(0:23, scan(), col="blue")

## 12 ##
c.K1 <- function (day) cos(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/23.93)
s.K1 <- function (day) sin(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/23.93)

c.O1 <- function (day) cos(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/25.82)
s.O1 <- function (day) sin(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/25.82)

c.M2 <- function (day) cos(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/12.42)
s.M2 <- function (day) sin(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/12.42)

c.S2 <- function (day) cos(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/12.00)
s.S2 <- function (day) sin(2*pi*(seq(0, 24, by=1/60) + 24*(day - 1))/12.00)

water.level <- z0 + a.K1 * c.K1(82) + b.K1 * s.K1(82) + a.O1 * c.O1(82) + b.O1 * s.O1(82) +
                    a.M2 * c.M2(82) + b.M2 * s.M2(82) + a.S2 * c.S2(82) + b.S2 * s.S2(82)
plot(seq(0,24,by=1/60), water.level, type="l")

water.level.fit <- matrix(
       c(c.K1(82), s.K1(82), c.O1(82), s.O1(82), c.M2(82), s.M2(82), c.S2(82), s.S2(82)),
        ncol=8) %*% coef(nagoya.fit)[-1] + coef(nagoya.fit)[1]
lines(seq(0,24,by=1/60), water.level.fit, col="red")

points(0:23, scan(), col="blue", type="b")
## END ##

computing software: R [available at http://www.r-project.org/ ]
Enjoy!
```

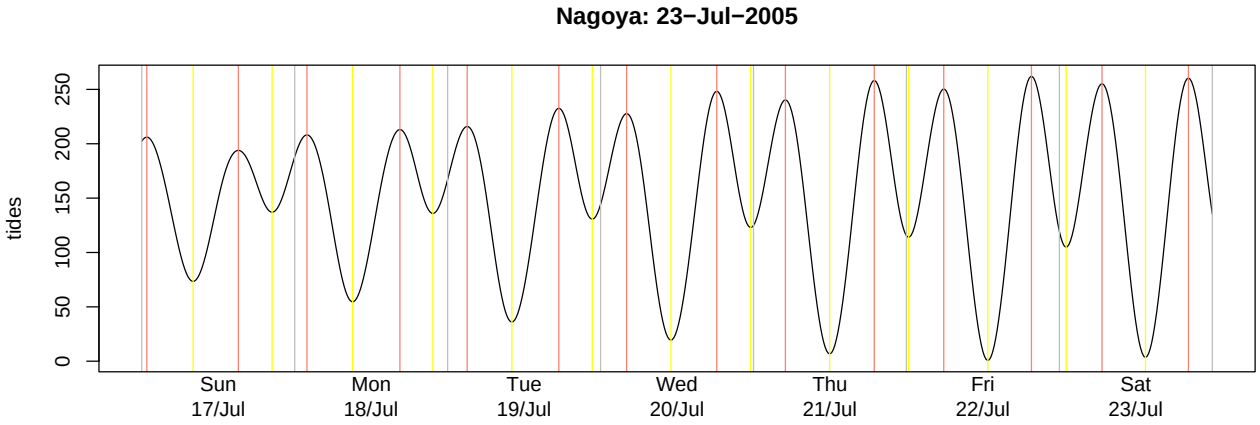
振動の成分を関数化

参考文献：

１）合田良実 (1998): 海岸・港湾，彰国社，321p.

２）宇野木早苗 (1993): 沿岸の海洋物理学，東海大学出版会，672p.

このスペースに
参考文献２）の表 3-1 （p. 80）




```
> tides.plot.week("Nagoya", "23-Jul-2005") # a package will be released in near future!
Based on the following 60 constituents:
 [1] "Sa"      "Ssa"     "Mm"      "MSf"     "Mf"      "2Q1"     "sigma1"  "Q1"
 [9] "rho1"    "O1"      "MP1"     "M1"      "chi1"    "pi1"     "P1"      "S1"
[17] "K1"      "psi1"    "phi1"    "theta1"  "J1"      "SO1"     "OO1"     "OQ2"
[25] "MNS2"    "2N2"     "mu2"     "N2"      "nu2"     "OP2"     "M2"      "MKS2"
[33] "lambda2" "L2"      "T2"      "S2"      "R2"      "K2"      "MSN2"    "KJ2"
[41] "2SM2"    "MO3"     "M3"      "SO3"     "MK3"     "SK3"     "MN4"     "M4"
[49] "SN4"     "MS4"     "MK4"     "S4"      "SK4"     "2MN6"    "M6"      "MSN6"
[57] "2MS6"    "2MK6"    "2SM6"    "MSK6"

High Water (cm):

 [1] "206.1/00:48/Sun" "193.9/15:10/Sun" "208.2/01:55/Mon" "213.1/16:31/Mon"
 [5] "215.9/03:04/Tue" "232.6/17:27/Tue" "227.7/04:05/Wed" "248.3/18:14/Wed"
 [9] "240.4/05:00/Thu" "258.2/18:57/Thu" "250.2/05:51/Fri" "261.9/19:37/Fri"
[13] "255.2/06:41/Sun" "260.4/20:15/Sun"

Low  Water (cm):

 [1] " 73.4/08:03/Sun" "  137/20:27/Sun" " 54.6/09:05/Mon" "135.9/21:41/Mon"
 [5] " 36.1/10:05/Tue" "130.6/22:42/Tue" " 19.4/11:03/Wed" "  123/23:34/Thu"
 [9] "  6.7/11:57/Thu" "114.1/00:21/Fri" "  0.7/12:47/Fri" "104.9/01:04/Sun"
[13] "  3.5/13:31/Sun"
>
```